

Tick your teacher

- Miss Cheng
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PERTH MODERN SCHOOL
YR11 MATHEMATICS SPECIALIST – 2018



PERTH MODERN SCHOOL
Exceptional schooling. Exceptional students.

TEST 1 – Reasoning (8%)

NAME: _____ **DATE:** Monday 26/02/18 7:45am

[To achieve full marks and to allow assessment of outcomes, working and reasoning should be shown.]
[A maximum of 2 marks will be deducted for incorrect rounding, units, etc.]

This is a *Calculator Free Assessment* – 45 minutes / 38 marks

1. [6 marks = 2, 2, 2]

Determine whether each of the following statement is true/false. Prove in general if the statement is true; disprove the false statements using counter-example(s).

(a) The sum of 3 consecutive whole numbers is divisible by 3.

(b) For any real number x , if x^2 is an odd number, then x must be an odd number.

(c) If a number is a multiple of m , and it is also a multiple of n , then it is a multiple of mn .

2. [3 marks]

If n is an integer, prove that $n + n^2$ is always even.

3. Prove the following inequality [4 marks]

$$\frac{a}{b} + \frac{b}{a} \geq 2$$

4. [4 marks]

Given that $X = 0.234343434\dots$. Convert X as a fraction.

5. [6 marks = 3, 3]

Write down the contrapositive of the following. Determine whether each of the contrapositive statements is true. Prove in general if the statement is true; disprove the false statements using counter-example(s).

(a) If a product of two positive real numbers is greater than 100, then at least one of the number is greater than 10.

(b) If $a, b \in R$, such that $a > b$, then $a^2 > b^2$.

6. [5 marks]

Use the fact that if n^2 is divisible by 5, then n is divisible by 5, to prove that $\sqrt{5}$ is irrational, using Proof by Contradiction.

7. [5 marks]

Use mathematical induction to prove that $4^{2n} - 1$ is always divisible by 5, for $n \in \mathbb{N}$.

8. [5 marks]

The total of adding up numbers that are doubled each time is the next term minus the first term. Verify this rule using proof by induction by proving the following.

$$\text{For } n \in \mathbb{N}, 5 + 10 + 20 + \dots + 5 \times 2^{n-1} = 5 \times 2^n - 5$$